(a) The frequency of oscillation f of a string depends on its length L, the force applied to its ends T, and the linear mass density ρ and is given as $f \propto L^{\alpha}T^{\beta}\rho^{\gamma}$. Using dimensional analysis, find the values of (a) $\alpha + \beta + \gamma$. (b) γ.
Ans: a) -1, b)-1/2

(b) A rigid body consists of 8 point masses sitting at the vertices of a regular octagon. Now one of its vertices are fixed to the origin (0,0,0) but allowing the octagon to rotate freely around the origin. How many degrees of freedom are left now ?

Ans: 3

(a) Consider the 3D-rotation matrix given below.

$$\begin{bmatrix} 1/3 & -2/3 & 2/3 \\ 2/3 & -1/3 & -2/3 \\ 2/3 & 2/3 & 1/3 \end{bmatrix}$$

Find a vector, among the options given below, which lies along the axis of this rotation.

(1) (1 0 -1)

(2) (1 0 0)

(3) (0 1 -1)

(4) (1 0 1)

Ans: 4

(a) If $u(x, y) = x + \frac{1}{2}(y^2 - x^2)$ is the real part of an analytic function $f(z) = u + iv$ of complex variable $z = x + iy$, then determine $v(x, y)$, the imaginary part of $f(z)$.
Ans: y(1-x)

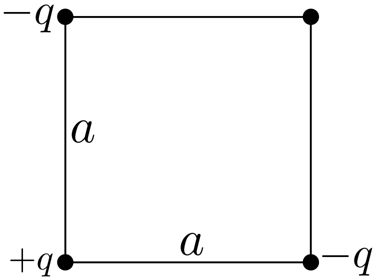
1. Solve the differential equation: $x\frac{d^2y}{dx^2} = \frac{dy}{dx}$ with boundary conditions, $y(1) = 0$ and $y'(1) = 1$.

Answer: (a) $y = (x^2 - 1)/2$

(a) Three charges are situated at the corners of a square (side a), as shown in the figure below. (a) The work done to bring in another charge, $+q$, from far away and place it in the fourth corner is given by $W = \frac{q^2}{4\pi\epsilon_0 a}B$, where B is a numerical value. What is the value of B.

(b) The dipole moment of the 4-charge configuration is $\vec{P} = Caq(\hat{x} - \hat{y})$. Determine C.

Ans: (a) $B = (-2 + \frac{1}{\sqrt{2}})$, (b) 0.



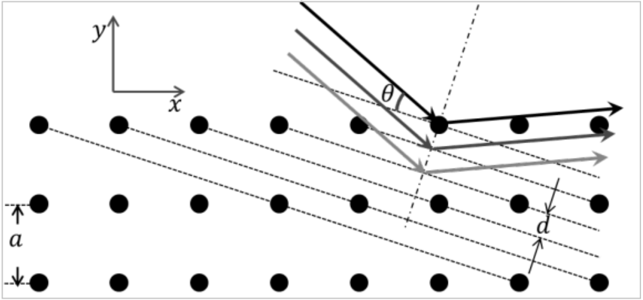
(a) An electric charge distribution produces an electric field $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}(1 - e^{-\alpha r})\frac{\hat{\mathbf{r}}}{r^2}$, where α is constant. Find the net charge within the radius (a) $R = \frac{1}{\alpha}$, and (b) at $R \to \infty$.

Answer: (a) $(1 - 1/e)$, (b) 0

(b) The electric field vector $\vec{E}$ of a monochromatic plane wave propagating along z-direction is given by $\vec{E} = E_0[\cos(kz - \omega t)\hat{x} - \frac{1}{\sqrt{2}}\sin(kz - \omega t)\hat{y}]$, where $\hat{x}$ and $\hat{y}$ are unit vectors along x- and y-directions respectively. Choose the correct type of polarization for this wave. Options are: (1) unpolarized, (2) plane (3) circular, (4) elliptical.

Ans: (4) elliptical

(a) X-rays of wavelength λ are diffracted by the atomic lines of a 2-dimensional (2D) crystal (equivalent to atomic planes of a 3D crystal), shown by the dotted lines in the Figure below. If the interatomic spacing (along both x and y directions) is a the angle $\theta = 45^o$. We can obtain $d = a/\sqrt{N_1}$ and $\lambda = a/\sqrt{N_2}$. Determine the values of, (1) N_1 and (2) N_2.



Ans: $N_1 = 10,\ N_2 = 5$.

(a) A particle of mass m is confined to a 3D potential, with harmonic traps on two sides (along y and z) and by an infinite wall along x. The potential is given by

$$\begin{aligned} V(x,y,z) &= \frac{1}{2}m\omega^2(y^2+z^2) \text{ for } \infty > y,z > -\infty \\ &= 0, \text{ for } a > x > 0 \end{aligned} \quad (1)$$

The 1st excited state can be three fold degenerate for a specific value of the frequency $\omega = N(\frac{\pi^2\hbar}{2ma^2})$. Determine N.

Ans: 3

(a) A Hydrogen atom is prepared in a superposition state, given by the wave function:

$$\Psi = \frac{1}{\sqrt{10}}[2\psi_{1,0,0} + \psi_{2,1,0} + \sqrt{2}\psi_{2,1,1} + \sqrt{3}\psi_{2,1,-1}] \tag{2}$$

where the subscripts denote the quantum numbers (n,l,m). Compute the energy expectation value of this state E_Ψ and express it as E_Ψ/E_{100}, where E_{100} is the energy of the lowest eigenstate.

Ans: $0.55E_{100}$

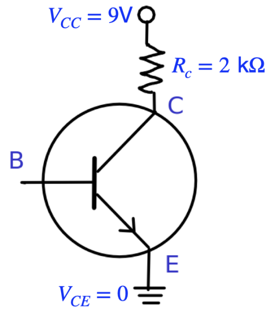
(a) One mole of ideal gas (obeying $PV = nRT$) is converted from the state-1 (with P_1, V_1, T_1) to state-2 (with P_2, V_2, T_2) isothermally (i.e., $T_2 = T_1$) and quasi-statically. The change of entropy $\Delta S = S_2 - S_1$ is given by the answer number :

(1) $nR\ln(V_1/V_2)$
(2) $nR\ln(P_1/P_2)$
(3) $nR\ln(P_2/P_1)$
(4) $nR\ln(P_1V_2/P_2V_1)$
Ans: 2

(a) A system of three spins S_1, S_2 and S_3, sitting on the vertices of a triangle, has energy $E = J(S_1S_2 + S_2S_3 + S_3S_1)$, with the constant $J > 0$. Each spin can take the values either $+1$ or -1. Find, (a) the degeneracy of the ground state and (b) entropy of the 1st excited state.

Ans: (a) 6, (b) k ln(2)

(a) A three resistor Π network consists of, a resistor "2R" between the input point and the ground, a second resistor "2R" between the output point and the ground and a third resistor "R" between the input point and the output point. If we demand that the effective input resistance and the output resistance of the circuit should be 12Ω each, then determine the value of "R" (in Ω).



Ans: 10 Ω